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Fundamental Interactions in the Early Universe

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A unified approach to fundamental interactions as explored earlier is further elaborated and its consequences for the early universe studied especially in connection with the energy dependence of the coupling constants.

In recent papers (Sivaram, 1993*a,b*) several interesting interrelationships between the coupling constants of the four fundamental interactions with underlying links to cosmological parameters were pointed out.

For instance, we have the unified relation

$$G_{\mathrm{F}} = \alpha^2 G_{\mathrm{N}} M_{\mathrm{Pl}}^2 \left(\frac{g_s^2}{\Lambda_{\mathrm{QCD}}}\right)^2 = \alpha^2 G_{\mathrm{N}} M_{\mathrm{Pl}}^2 \alpha_s^2 \left(\frac{\hbar c}{\Lambda_{\mathrm{QCD}}}\right)^2 \tag{1}$$

Here G_{F} is the universal Fermi weak interaction constant ($\simeq 1.4 \times 10^{-49}$ erg cm^3); G_{N} is the Newtonian gravitational constant ($\simeq 6.67 \times 10^{-8}$ cgs units); $\alpha \simeq 1/137$ is the electromagnetic fine structure constant; $M_{\mathrm{Pl}} = (\hbar c/G_{\mathrm{N}})^{1/2}$ is the Planck mass; $\alpha_s = g_s^2/\hbar c \approx 0.12$ is the strong quark–gluon coupling constant; $\hbar$ and c are Planck's constant and the velocity of light, respectively; $\Lambda_{\mathrm{QCD}} \simeq 180$ MeV is the QCD energy scale. In Sivaram (1993*b*), this equation was also written as $G_{\mathrm{F}} = \alpha^2 G_{\mathrm{N}} M_{\mathrm{Pl}}^2 (\hbar/2M_P c)^2$, M_P being the proton mass.

As is well known, grand unified theories (GUT) of strong and electroweak interactions at lower energies reduce to effective four-fermion interactions mediated by exchanges of superheavy bosons (mass M_X) with an 'effective Fermi constant' given by

$$G_{\mathrm{GUT}} \simeq G_{\mathrm{F}} \left(\frac{m_W}{M_X}\right)^2 \approx 2 \times 10^{-75} \text{ erg cm}^3 \tag{2}$$

(m_W is the weak boson mass ~ 100 GeV).

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Again, analogous to the weak interaction case, where we have

$$g_w^2/m_W^2 \simeq G_F \tag{3}$$

here we have

$$g_{GUT}^2/M_X^2 \simeq G_{GUT} \tag{4}$$

Again in earlier works (Sivaram, 1979, 1982, 1986*a*,*b*, 1993*b*) it was pointed out that the Dirac equation for interacting fermions in a curved space with background torsion has an effective four-fermion self-interaction nonlinear spinor term of the form

$$\gamma^\mu \psi_{;\mu} + \left(\frac{3}{8}\hbar G_{eff}/c^3\right)\gamma^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi)\gamma^5\psi = 0 \tag{5}$$

where the effective strong gravitational constant G_{eff} is related to the effective four-fermion constant G_{Feff} by

$$G_{eff} = G_{Feff} \cdot c^2/\hbar^2 \tag{6}$$

So in this picture all fundamental interactions arise through four-fermion self-interactions in a curved space with torsion (non-Riemannian or Riemann–Cartan space), the curvature and torsion giving rise to such interaction terms. Spin–torsion interaction terms arising from such a background space (for the fundamental fermions present in it) give rise to the observed effective interactions, including gravity.

This interaction constant G_{eff} or G_{Feff} has its value depending on the energy scale at which the interaction is switched on, i.e., there is only one unified coupling which scales with energy as G_{eff} (or G_{Feff}) $\sim 1/E^2$. In Sivaram (1990) this scaling was understood in terms of an energy-dependent string tension, T scaling as E^2 (i.e., $T \to c^2/G_{eff}$). At the Fermi beta decay scale ($E \sim 10^2$ GeV), $G_{eff} = G_F c^2/\hbar^2$, at the Planck scale ($E \sim 10^{19}$ GeV), $G_{eff} = G_N$, the usual Newtonian constant [or $(G_F)_g = G_N\hbar^2/c^2 = 6 \times 10^{-83}$ erg cm^3], at the QCD scale, $G_{eff} = G_f$ (the strong gravity constant $\approx 10^{38}$ G_N), giving an effective interaction strength $g^2/\hbar c \sim 1$, and so on. Again as shown in Sivaram (1990, 1993*b*), the different strengths of the various interactions as measured by $(G_F)_{eff}$ arises from the distribution of the same universal quantum coupling constant $\beta\hbar c$ ($\beta \sim 1$) distributed over regions of space-time of different surface areas. Thus the product ($\beta\hbar c$ × area) gives $(G_F)_{eff}$. If the area is the square of the beta-decay length $(G_F/\hbar c)^{1/2}$, then $\beta\hbar c$ × area = G_F, the Fermi β-decay constant. If the area is the square of the Planck length, then $\beta\hbar c$ × Planck area = $(G_F)_g$ giving the Newtonian constant G_N [through equation (6)]. For strong interactions, the area is the proton Compton length squared, etc.

In general

$$\beta\hbar c \times \text{area} = (G_{\text{F}})_{\text{eff}} \tag{7}$$

From the uncertainty principle [also explained by energy-dependent string tension as in Sivaram (1990)]

$$\text{area} \propto \frac{1}{E^2}, \qquad (G_{\text{F}})_{\text{eff}} \propto \frac{1}{E^2} \quad \text{or} \quad \frac{1}{M^2} \tag{8}$$

where E or M is the energy scale and

$$\beta\hbar c = g_{\text{eff}}^2 \tag{9}$$

is a universal constant characterizing the unified interaction strength. Thus

$$\frac{g_{\text{eff}}^2}{M^2} \simeq (G_{\text{F}})_{\text{eff}} \tag{10}$$

thus explaining equations (3) and (4).

Equation (7) [giving rise to equation (10)] can also be elaborated through a recent work (Sivaram, 1993*c*,*d*) where it was shown that for a torsion-supported self-gravitating system the equilibrium radius is given as

$$R \approx \left(\frac{G_{\text{eff}}\hbar^2}{Mc^4}\right)^{1/3} \times \text{const} \tag{11}$$

This gives

$$Mc^2R^3 \simeq G_{\text{eff}}\hbar^2/c^2 = (G_{\text{F}})_{\text{eff}} \tag{12}$$

[through equation (6)] or

$$ER^3 = (G_{\text{F}})_{\text{eff}} \tag{13}$$

which can be written ER : area $= (G_{\text{F}})_{\text{eff}}$ (as area $\propto r^2$).

From the uncertainty principle $ER \simeq \hbar c$, giving finally $\beta\hbar c \times \text{area} \approx (G_{\text{F}})_{\text{eff}}$, which is the same ($\beta \approx 1$) as equation (7). Essentially equation (13) suggests that energy E squeezed into a volume V gives a $(G_{\text{F}})_{\text{eff}}$ that is dependent on energy owing to quantum effects. As in equation (13), $R \propto 1/E$ (for quantum systems), $(G_{\text{F}})_{\text{eff}}$ is proportional to $1/E^2$ or $1/M^2$, or defining $\beta\hbar c$ as g_{eff}^2 [equation (9)], we have $g_{\text{eff}}^2 = (G_{\text{F}})_{\text{eff}}E^2$. Now in equation (10), it is assumed that g_{eff} is a constant with energy and only $(G_{\text{F}})_{\text{eff}}$ scales as $1/M^2$ (since $g_{\text{eff}}^2 = \beta\hbar c$ is a universal constant). Since

$$g_{\text{eff}}^2 = (G_{\text{F}})_{\text{eff}} \cdot M^2 = \text{const} \tag{14}$$

However, in the usual GUT scenario, g_{eff} also slowly grows with energy as $\log M$. Thus the renormalization group equation

$$M(dg^2/dM) = 2/3\pi(g^2)^2$$

yields

$$g^2(M_0) = \frac{g^2(M)}{1 + (2/3\pi)g^2(M)\ln(M/M_0)} \tag{15}$$

or

$$g^2_{\rm eff} \approx g^2_0/\ln(M/M_0) \tag{16}$$

or from equation (10)

$$g^2_{\rm eff} \simeq (G_{\rm F})_{\rm eff}\frac{M^2}{\ln(M/M_0)} \tag{17}$$

where

$$(G_{\rm F})_{\rm eff} = (G_{\rm F})_0\left(\frac{M_0}{M}\right)^2 \tag{18}$$

[also $G_{\rm eff} = G_0(M_0/M)^2$].

While the torsion of the background space was seen to give rise to relations like equations (10) and (18), the logarithmic variation with energy of $g^2_{\rm eff}$ can be considered as an effect of the curvature of space. It has been shown by Calzetta *et al.* (1985) that in a general curved space-time the effective value of the gauge coupling constant $g^2_{\rm eff}$ can decrease logarithmically with curvature K as

$$g^2_{\rm eff} \simeq \frac{g^2_0}{(\ln K/K_0)^{-1/2}} \tag{19}$$

which translates into $g^2_{\rm eff} \simeq g^2_0/\ln(M/M_0)$ as in equation (16) (as $M^2 \propto 1/K$, $M \propto 1/\text{radius of curvature}$). Thus we naturally get equation (17), where $(G_{\rm F})_{\rm eff}M^2 \approx \text{const} \approx \beta\hbar c$ ($\beta \approx 1$). Asymptotic freedom is implied in both equations (16) and (19), as $M \to \infty$, $K \to \infty$ gives $g^2_{\rm eff} \to 0$. This implies that the unified coupling constant tends to become vanishingly small in the early universe, when energies and curvatures become infinitely large, i.e., $(G_{\rm F})_{\rm eff} \to 0$, $g^2_{\rm eff} \to 0$, and $G_{\rm eff} \to 0$.

At Planck energies $G_{\rm eff} \to G_{\rm N}$, $g^2_{\rm eff} \to \hbar c$ (gravity behaves like a strong interaction). Equations (10)–(19) imply that all energy variations of the effective coupling constant [$g^2_{\rm eff}$ or $(G_{\rm F})_{\rm eff}$] arise from the geometrical effects of torsion and curvature of the background space.

In Sivaram (1990, 1993*b*) it was argued that below $E_{\rm Pl}$, equations (18)–(19) hold and string tension $T = c^2/G_{\rm eff}$ scales as E^2 [equation (18)]. This accounted for the strong electroweak and gravitational couplings. A space-based experiment to test this was also suggested in Sivaram (1993*b*). At $E > E_{\rm Pl}$ it was argued that T and $G_{\rm eff}$ remain constant (Sivaram, 1990, 1993*b*) while $(G_{\rm F})_{\rm eff} \to G_{\rm eff}\cdot\hbar^2/c^2$ scales as $1/M^4$ (M scales as R) since $\hbar$

scales as $1/M^2$. Thus the effective coupling $g_{\rm eff}^2 \to \beta\hbar c$ vanishes as $M \to \infty$ or $R \to 0$ ($K \to \infty$). In fact above $E > E_{\rm Pl}$ the gravitational action is replaced by a Born–Infeld type of theory nonlinear in curvature and expanded as (Sivaram, 1986*a*,*b*)

$$L_{\rm eff} = \int d^4x(-g)^{1/2}\left[\frac{R}{16\pi G} + \hbar R^2 + \frac{1}{c}(G_{\rm F})_{\rm eff}R^3 \ldots\right]$$

This has solutions with no singular behavior as $M \to \infty$. At $E < E_{\rm Pl}$ one has the usual Hilbert action for gravity (Sivaram, 1986*a*, 1992*a*).

It must also be remarked that equation (11), which arises as an exact solution of the Einstein–Cartan theory modified to include internal quantum numbers, has a wide applicability. (It has a 3/8 factor inside brackets) (Sivaram, 1979). For $M = m_e$ the electron mass, and $G_{\rm eff} - G_f = G_{\rm N} \cdot 10^{38}$ (the strong gravity constant), it gives

$$R = \left(\frac{3G_f\hbar^2}{8m_e c^4}\right)^{1/3} = 1.4 \times 10^{-13}\,\text{cm} \tag{20}$$

(the classical electron radius).

With $M = m_P$ the proton mass, and $G_{\rm eff} = G_f$, equation (20) gives the proton Compton length, i.e., $R \simeq 10^{-14}$ cm. With $M = M_W$ the weak boson mass and $G_{\rm eff} = G_{\rm F} \cdot c^2/\hbar^2$ it gives the β-decay length, i.e., $R \simeq 7 \times 10^{-17}$ cm. With $M = M_{\rm Pl}$ the Planck mass and $G_{\rm eff} = G_{\rm N}$ it gives the Planck length $R \simeq 10^{-33}$ cm, etc. The 3/8 factor would imply a $g_0^2 \simeq \beta\hbar c$ with $\beta \approx 0.1$ for quarks and 0.3 for gluons.

The relation $ER^3 \simeq (G_{\rm F})_{\rm eff} \simeq G_{\rm eff} \cdot \hbar^2/c^2$ [see equation (13)] also has implications for macroscopic (astronomical) objects where $\hbar$ is replaced by the intrinsic angular momentum or action J of the system. (For such systems of course $G_{\rm eff} = G_{\rm N}$.)

Thus we can write (Sivaram, 1993*b*)

$$ER^3 \approx \beta G_N J^2/c^2 \tag{21}$$

As examples: For a neutron star $J \simeq 10^{50}$ cgs, $E \simeq 10^{54}$ ergs and $R \simeq 10^6$ cm, and for the universe (Sivaram, 1993*b*) $J \simeq 10^{93}$ cgs, $E \approx 10^{77}$ ergs, and $R \simeq 10^{28}$ cm and equation (21) is satisfied. It also holds for other systems if c is replaced by a typical velocity characteristic of the system. Interestingly $\beta G_{\rm N}/c^2$ is the inverse of the superstring tension (Sivaram, 1987). In Sivaram (1993*b*), it was shown how $(G_{\rm F})_{\rm eff}$ as defined above for macroscopic bodies implies magnetic moments for such systems. We again have a framework for effectively describing a whole hierarchy of scales in the universe from the Planck length to the Hubble radius.

The behavior of the effective coupling at $E < E_{\rm Pl}$ is also consistent with the model given in Sivaram (1986*a*,*b*, 1987*a*,*b*, 1990). There it was suggested that at energies $>E_{\mathrm{Pl}}$ the action must be described by a quantity quadratic in the field strength (equivalently curvature) which is invariant under both general coordinate transformations and local scale transformations (ST) and which has a dimensionless running coupling constant α_V. A suitable action which is renormalizable and asymptotically free with all these properties is the Weyl action

$$\alpha_V \int d^4\chi(-g)^{1/2} C^{abcd} C_{abcd} \tag{22}$$

The action (22) was used in another context in a strong gravity version of QCD which is a scale invariant and renormalizable theory (Salam and Sivaram, 1993).

Here equation (22) describes 'gravity' which also includes all interactions at $E > E_{\mathrm{Pl}}$, i.e., at such energies we have only one unified interaction with a dimensionless coupling α_V, which is asymptotically free, $\alpha_V \to 0$, at $E \to \infty$. As shown in the above papers, the Weyl action implies $M \propto R$, i.e., confinement of energy, like the linearly rising potential confining color in QCD. As argued in Sivaram (1990), this implies a constant string tension ($T = c^2/G_{\mathrm{eff}}$, $G_{\mathrm{eff}} = G_{\mathrm{N}} = \mathrm{const}$) and energy $\approx T \times R \propto R$. Thus $GM^2_{\mathrm{eff}} \propto R^2$, i.e. $g^2_{\mathrm{eff}} = \beta\hbar c \approx G_{\mathrm{eff}} M^2$ scales as R^2, vanishing at $R \to 0$, which implies asymptotic freedom. Thus the interactions tend to vanish as $R \to 0$. The action (22) for the unified description of gravity and GUTs at energies $>E_{\mathrm{Pl}}$ can also be written as (Sivaram, 1987*a,b*)

$$\alpha_V \int \alpha_V d^4\chi(-g)^{1/2}\left(W^2 - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}\right) \tag{23}$$

where W is the Weyl curvature scalar, related to the Riemann scalar as

$$W = R - 6(A^\mu, \mu - A^\mu A_\mu) \tag{24}$$

where A is the Weyl four-vector gauge transforming as $A^a_\mu \to A^a_\mu + \lambda, \mu$, with λ the scale parameter. The index a is the Yang–Mills field strength and can take values depending on the groups and multiplets considered. Scale invariance can be broken (there is no energy scale, $\alpha_V \to 0$ as $E \to \infty$) at the Planck scale, where we can set $W = \Lambda_{\mathrm{Pl}}$. Using the relation connecting W and R and the transformations $A'_\mu = (\Lambda_{\mathrm{Pl}})^{-1/2} A_\mu$ and $F'_{\mu\nu} = (\Lambda_{\mathrm{Pl}})^{-1/}$ $2F_{\mu\nu}$, we have a natural separation of gravity and GUTs below E_{Pl}

$$\delta \int (-g)^{1/2}(\kappa R + \alpha_{\mathrm{GUT}} F'^a_{\mu\nu} F'^{a\mu\nu} - \Lambda_{\mathrm{Pl}}(\tfrac{1}{2} + 6A'_\mu A'^\mu) = 0$$

with κ a *dimensional* constant now related to Λ_{Pl} and fixing the Newtonian G_{N}. Including the fundamental fermionic sector, through actions leading to equation (5), one gets a coupling of the effective α_{GUT} to the torsion and curvature of the space-time leading to the scaling relations governed by equations (10)–(19).

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